

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1

Branch: Diploma (All)

Semester : 2

Date : 20/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  - (2) Instructions written on main answer book are strictly to be obeyed.
  - (3) Draw neat diagrams and figures (if necessary) at right places.
  - (4) Assume suitable data if needed.
- 

**Q-1**                      **Attempt the following questions:**                      **(14)**

- a) The distance between the points (2, 3) and (3, 4) is \_\_\_\_\_.  
(A)  $\sqrt{2}$  (B)  $2\sqrt{2}$  (C) 25 (D) None of these
- b) If A(-7, 2) and B(3, 8) then mid-point of AB = \_\_\_\_\_.  
(A) (-2,5) (B) (5,-2) (C) (2,5) (D) (5,2)
- c) x – intercept of line  $3x + 2y - 7 = 0$  is \_\_\_\_\_.  
(A)  $7/2$  (B)  $-7/2$  (C)  $7/3$  (D)  $-7/3$
- d) Centre of the circle  $x^2 + y^2 = 25$  is \_\_\_\_\_.  
(A) (0,5) (B) (5,0) (C) (0,0) (D) None of these
- e)  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{x}{2}} =$  \_\_\_\_\_.  
(A)  $e^2$  (B)  $e$  (C)  $e^{1/2}$  (D) None of these
- f)  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{5\theta} =$  \_\_\_\_\_.  
(A) 1 (B) 0 (C)  $1/5$  (D) 5
- g)  $\lim_{x \rightarrow 0} \frac{x^2 + x + 1}{x + 1} =$  \_\_\_\_\_.  
(A) -1 (B) 0 (C) 1 (D) None of these
- h)  $\frac{d(\sec x)}{dx} =$  \_\_\_\_\_.  
(A)  $\tan x$  (B)  $\tan^2 x$  (C)  $-\tan x$  (D)  $\sec x \tan x$
- i)  $\frac{d(1/x)}{dx} =$  \_\_\_\_\_.  
(A)  $-\frac{1}{x^2}$  (B)  $\frac{1}{x}$  (C)  $e$  (D) 1



- j)  $\frac{d(\sin^{-1} x + \cos^{-1} x)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $\frac{\pi}{2}$  (B)  $-1$  (C)  $0$  (D)  $1$
- k)  $\frac{d(x^2 + 2x + 7)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $2x$  (B)  $2x + 1$  (C)  $2x + 2$  (D) None of these
- l)  $\int e^x dx = \underline{\hspace{2cm}}$   
 (A)  $\log x + c$  (B)  $e^x + c$  (C)  $1$  (D)  $0$
- m)  $\int a^x dx = \underline{\hspace{2cm}}$   
 (A)  $\frac{a^x}{\log_e a} + c$  (B)  $a^x \log_e a + c$  (C)  $a^x + c$  (D)  $\log_e a + c$
- n)  $\int_2^5 x^3 dx = \underline{\hspace{2cm}}$   
 (A)  $\frac{641}{4}$  (B)  $\frac{609}{4}$  (C)  $\frac{690}{4}$  (D)  $\frac{614}{4}$

**Attempt any four questions from Q-2 to Q-8**

- Q-2 Attempt all questions (14)**
- a) In which ratio  $X$  – axis divides line segment joining points  $(2, 2)$  and  $(-3, 6)$ ? Find coordinates of division point. (5)
- b) Find the equation of perpendicular bisector to line joining points  $(-1, 2)$  and  $(1, -2)$ . (5)
- c) If  $f(x) = \frac{ax+b}{bx+a}$  then prove that  $f(x) \cdot f\left(\frac{1}{x}\right) = 1$ . (4)
- Q-3 Attempt all questions (14)**
- a) Prove that  $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x} = \frac{1}{\sqrt{a}}$ . (5)
- b) Evaluate:  $\lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos x}$  (5)
- c) Prove that  $(12, 8)$ ,  $(-2, 6)$  and  $(6, 0)$  are the vertices of an isosceles right angled triangle. (4)
- Q-4 Attempt all questions (14)**
- a) Find centre and radius of circle  $2x^2 + 2y^2 - 8x + 4y + 2 = 0$ . (5)
- b) Find derivative of  $f(x) = x^3 - 2x$  using definition. (5)
- c) If  $y = \frac{\log x}{x}$  then find  $\frac{dy}{dx}$  at  $x = 1$ . (4)
- Q-5 Attempt all questions (14)**
- a) Evaluate:  $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^2(1 + 2 + \dots + n)}$  (5)



b)  $S = t^3 - 6t^2 + 9t + 6$  gives the distance travelled by a body in  $t$  seconds. (5)  
Find velocity and acceleration at  $t = 4$  seconds.

c) Find  $\frac{dy}{dx}$  if  $y = (e^{3x} + 1)^{2x+5}$ . (4)

**Q-6**

**Attempt all questions** (14)

a) Evaluate:  $\int \left[ \sqrt{1 + \sin 2x} + \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}} \right] dx$  (5)

b) If  $y = 2e^{-3x} + 3e^{2x}$  then prove that  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ . (5)

c) Evaluate:  $\int x\sqrt{x^2 - a^2} dx$  (4)

**Q-7**

**Attempt all questions** (14)

a) Find  $\frac{dy}{dx}$  if  $y = \log \sqrt{\frac{a+x}{a-x}}$ . (5)

b) Evaluate:  $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx$  (5)

c) Evaluate:  $\int x^n \log x dx$  (4)

**Q-8**

**Attempt all questions** (14)

a) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$ . (5)

b) Find the area of the region bounded between curve  $y = x^2$  and straight – line  $x = 2$ . (5)

c) If the radius of a circle  $x^2 + y^2 - 4x - 8y + k = 0$  is 4, find  $k$ . (4)

